ANALYTICAL ANALYSIS OF NON LINEAR DYNAMIC MOTION OF SPUR GEAR PAIR EXCITATION

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ABSTRACT

Gears are widely used in almost each type of machineries in the industry. Along with bolts, nuts and screws; they are a common element in machines and will be needed frequently by machine designers to realize their designs in almost all fields of mechanical applications. These toothed wheels were used to transmit circular motion or rotational force from one part of a machine to another. Gears are used in pairs and each gear is usually attached to a rotating shaft. Spur gears consist of a cylinder or disk with the teeth projecting radically, and although they are not straight-sided in form, the edge of each tooth is straight and aligned parallel to the axis of rotation. Spur gears have a wide range of applications.

Dynamic analysis of geared systems is an essential step in design due to two reasons. First, under the driving conditions, a typical geared system is subject to dynamic forces which can be large. Therefore, the prediction of dynamic loads, motions or stresses is needed in developing reliable gear trains. Second, the vibration level of the geared system is directly related to the noise radiated from the gear box. An attempt in designing quiet gears requires a good understanding of the dynamic behavior of the system and the gear mesh source.

The internal excitation is of importance from the high frequency noise and vibration control viewpoint and it represents the overall kinematic or static transmission error. Such problems may be significantly different from the rattle problems associated with external, low frequency torque excitation

INTRODUCTION

1.1 GENERAL INTRODUCTION

This analytical study on the non-linear dynamics of a spur gear pair with backlash as excited by the static transmission error has made a number of contributions to the state of the art. A non-linear system does not satisfy the superposition principle .Thus the output of a nonlinear system is not directly proportional to the input.

1.2 History of Gearing

Gears are widely used in almost each type of machineries in the industry. Along with bolts, nuts and screws; they are a common element in machines and will be needed frequently by machine designers to realize their designs in almost all fields of mechanical applications. Ever since the first gear was conceived over 3000 years ago, they have become an integral component in all manner of tools and machineries. The earliest gear drives were crude and used rods inserted in one wheel meshing with identical rods mounted axially in an another wheel as shown in Figure 1.1. These toothed wheels were used to transmit circular motion or rotational force from one part of a machine to another. Gears are used in pairs and each gear is usually attached to a rotating shaft.

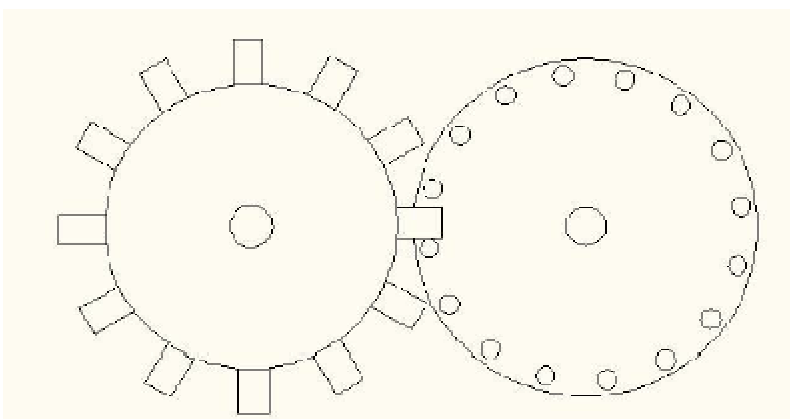


Figure 1.1: Sketch of early gear system [1]

Although ineffective, this type of gear drive performed satisfactorily at low speeds and loads. The main trouble with this system was encountered when the loads and speeds were raised. The contact between the rods were in effect a point contact, giving rise to very high stresses which the materials could not withstand and the use of any lubrication was obsolete due to the contact area, hence high wear was a common occurrence. Although not so obvious at that time, the understanding of the speed ratio of the gear system was critical. Due to the crude design of the system, the speed ratio was not constant.

As a result, when one gear ran at constant speed, there was regular acceleration and deceleration of each tooth of the other gear. The loads generated by the acceleration influence the steady drive loads to cause vibration and ultimately failure of the gear system. Since the 19th century the gear drives designed have mainly been concerned with keeping contact stresses below material limits and improving the smoothness of the drive by keeping velocity ratios as constant as possible. The major rewards of keeping the velocity ratio constant is the reduction of dynamic effects which will give rise to increase in stress, vibration and noise.

Gear design is a highly complicated skill. The constant pressure to build cheaper, quieter running, lighter and more powerful machinery has given rise to steady and advantageous changes in gear designs over the past few decades.

1.3 A Spur Gear

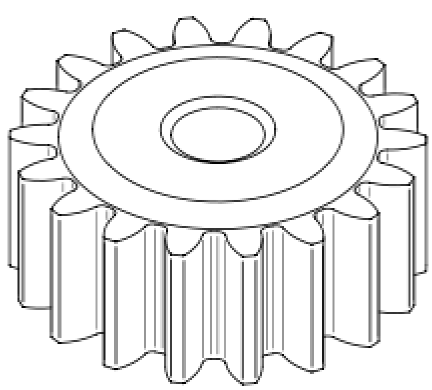


Figure 1.2: A Spur Gear [14]

Spur gears are the simplest type of gear. They consist of a cylinder or disk with the teeth projecting radically, and although they are not straight-sided in form, the edge of each tooth is straight and aligned parallel to the axis of rotation. These gears can be meshed together correctly only if they are fitted to parallel shafts.

Spur gears have a wide range of applications. They are used in:

Metal cutting machines, Power plants, Marine engine, Mechanical clocks and watches, Fuel pumps, Washing Machines, Gear motors and gear pumps, Rack and pinion mechanisms, Material handling equipments, Automobile gear boxes, Steel mills, Rolling mills etc.

1.4 Excitation Types and Backlash

The focus of this segment is on the backlash non-linearity as excited primarily by the transmission error between the spur gear pair. A gear pair is bound to have some backlash which may be either designed to provide adequate lubrication and eliminate interference due to manufacturing errors. Backlash induced torsional vibrations may cause tooth separation and impacts in unloaded or lightly loaded geared drives. Such impacts result in intense vibration and noise problems and large dynamic loads, which may affect reliability and life of the gear drive [3, 4]. Excitation mechanisms can be grouped as follows:

1.4.1 External Excitations:

This group includes excitations due to rotating mass unbalances, geometric eccentricities, and prime mover and load torque fluctuations [5]. Although mass unbalances and geometric eccentricities can be reduced through improved design and manufacturing, torque fluctuations are not easy to eliminate since they are determined by the characteristics of the prime mover (piston engines, dc motors etc.) and load [6]. Such excitations are typically at low frequencies Ω_T which are the first few multiples of the input shaft speed Ω_S . Practical examples include rattle problems in lightly loaded automotive transmissions and machine tools [6, 7].

1.4.2 Internal excitations:

This group includes high frequency Ω_h excitations caused by the manufacturing related profile and spacing errors, and the elastic deformation of teeth, shafts and bearings. Under the static conditions, all such mechanisms can be combined to yield an overall kinematic error function known as "the static transmission error" e (t) [4, 5]. This error is defined as the difference between the actual angular position of the driven gear and where it would be if the gears were perfectly conjugate [4, 5, 8-10]. In gear dynamic models, e (t) is modeled as a periodic displacement excitation at the mesh point along the line of action [2, 11-13] and its period is given by the fundamental meshing frequency. Ω_h =NΩ_S, where N is the number of teeth on the pinion. Practical examples include steady state noise and vibration problems in automotive, aerospace, industrial, marine and appliance geared systems.

1.5 Problem Formulation

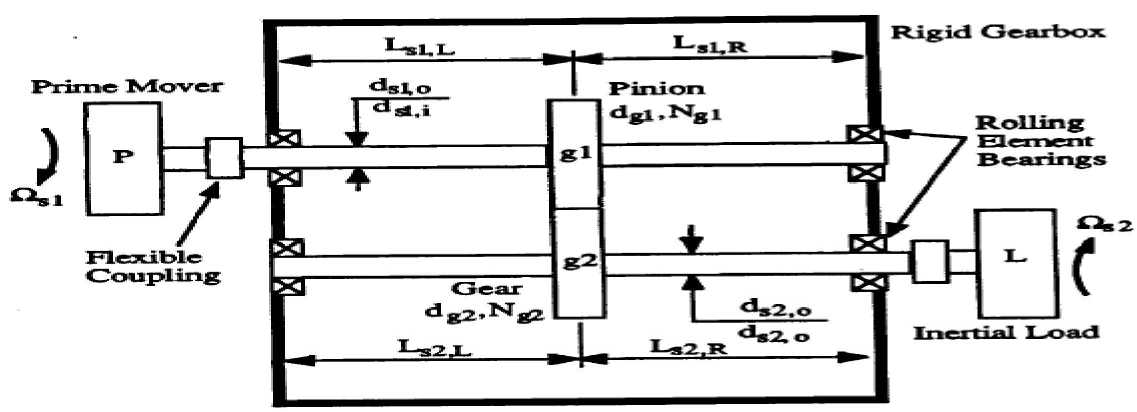
Dynamic analysis of geared systems is an essential step in design due to two reasons. First, under the driving conditions, a typical geared system is subject to dynamic forces which can be large. Therefore, the prediction of dynamic loads, motions or stresses is needed in developing reliable gear trains. Second, the vibration level of the geared system is directly related to the noise radiated from the gear box. An attempt in designing quiet gears requizes a good understanding of the dynamic behavior of the system and the gear mesh source. Accordingly, the main objective of this study is to develop accurate mathematical models of a generic geared rotor-bearing system shown in Figure 1.2(a). Of interest here is to investigate several key modelling issues which have not been addressed in the literature, such as system non-linearities and time-varying mesh stiffness.

The generic geared system shown in Figure 1.2(a) consists of a single spur gear mesh of ratio Vg=dg_2/ dg_{21} rolling element bearings, a prime mover driving the system at Ω_{s1} speed and a typical inertial load. The system also includes other elements such as couplings and flywheel. A discrete model of the system is shown in Figure 1.2(b). Here, shafts are represented by discrete translational spring's k_{s1} andk_{s2}, translational dampers C_{s1} andC_{s2}, and torsional springs k_{s1} andk_{s2}. The gear mesh is represented by time-varying mesh stiffness k_h (t) and a non-linear displacement function f_h which includes gear backlash. Further, linear time-invariant (LTI) mesh damping C_h is considered here. The rolling element bearings are defined by a time-invariant radial stiffness K_b subject to a non-linear displacement functionf_b, and an LTI damping coefficientC_b. The prime mover and load are modeled as purely torsional elements of in ertias I_P, andI_L, respectively. The mean rotational speeds Ω_{S1} and Ω_{S2} and the geometric end conditions are such that gyroscopic effects are not seen.

The generalized displacement vector {q(t)}, associated with the inertia elements, consists of angular displacements 0 and transverse displacements ‘x’ and ‘y’. The governing equation of motion for the non-linear, time-varying multi-degree of freedom model can be given in the general form as

$$\mathbf{[M]\{q''(t)\}+[C]\{q'(t)\}+[K(t)]\{f(q(t))\}=[F(t)]} \tag{1.1}$$

where [M] is the time-invariant mass matrix and {q(t)} is the displacement vector. Here, damping matrix [C] is assumed to be LTI type, as the effect of the tooth separation and time-varying mesh properties on mesh damping are considered negligible; validity of this assumption will be examined later. The stiffness matrix [K(t)] is considered to be time-varying, given by a periodically time varying matrix [K(t)] = [K(t+2π / Ω_h)] where Ω_h is the fundamental gear mesh frequency. The non-linear displacement vector {f(q(t))} includes the radial clearances in bearings and the gear backlash, and the forcing vector {F(t)} consists of both external excitations due to torque fluctuations, mass unbalances and geometric eccentricities, and an internal static transmission error excitation.



(a)

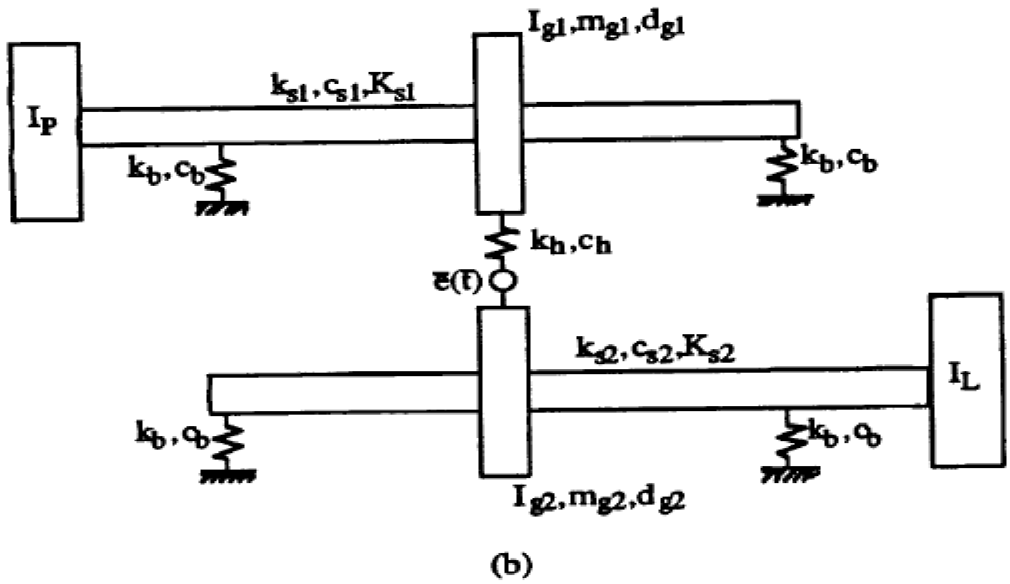


Figure 1.3: a) A generic geared rotating system, b) discrete model of geared rotating system. [2]